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## **$c \rightarrow u\gamma$ in Cabibbo suppressed D meson radiative weak decays**

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### **ABSTRACT**

We investigate Cabibbo suppressed  $D^0$ ,  $D^+$  and  $D_s^+$  radiative weak decays in order to find the best mode to test  $c \rightarrow u\gamma$  decay. Combining heavy quark effective theory and the chiral Lagrangian approach we determine the decay widths. We calculate  $\Gamma(D^0 \rightarrow \rho^0/\omega\gamma)/\Gamma(D^0 \rightarrow \bar{K}^{*0}\gamma)$  previously proposed to search for possible New Physics. However, we notice that there are large, unknown, corrections within the Standard Model. We propose a better alternative, the ratio  $\Gamma(D_s^+ \rightarrow K^{*+}\gamma)/\Gamma(D_s^+ \rightarrow \rho^+\gamma)$ , and show that it is less sensitive to the Standard Model.

## 1 Introduction

According to the Standard Model, the physics of charm mesons is not as exciting as the physics of bottom mesons [1, 2, 3]: the relevant CKM matrix elements  $V_{cs}$  and  $V_{cd}$  are well known, the  $D^0 - \bar{D}^0$  oscillations and CP asymmetries are small, weak decays of D mesons are difficult to investigate due to the strong final state interactions, and very small branching ratios are expected for rare decays. However, authors [1, 2, 3] have noticed that the  $D^0 - \bar{D}^0$  oscillation and  $c \rightarrow u\gamma$  decays obtain contributions coming from non-minimal supersymmetry which are not present within the Standard Model. Therefore, these two phenomena might be guides for a signal of New Physics. In ref. [2] it was observed that New Physics can generate  $c \rightarrow u\gamma$  transitions leading to a deviation from

$$R_{\rho/\omega} \equiv \frac{\Gamma(D^0 \rightarrow \rho^0/\omega\gamma)}{\Gamma(D^0 \rightarrow \bar{K}^{*0}\gamma)} = \frac{\tan^2\theta_c}{2} \quad (1)$$

(the factor  $\frac{1}{2}$  was overlooked in refs. [1, 2]). Motivated by the importance of this signal we investigate Cabibbo suppressed radiative weak decays in which  $c \rightarrow u\gamma$  transition occurs. As a theoretical framework we use a hybrid theory: a combination of heavy quark effective theory (HQET) and chiral Lagrangians (CHL) [4, 5, 6, 7, 8]. This approach, accompanied by the factorization hypothesis, enables us to use the Standard Model results for electroweak processes. It is possible to apply other approaches like for example [9], but the result which indicates the deviation from  $\tan^2\theta_c$  cannot be very different from ours obtained with HQET + CHL. In fact, our results agree with [9] within the uncertainties.

We calculate the ratios between various Cabibbo suppressed and Cabibbo allowed charm meson radiative weak decays. Analysing them we notice that the relation (1) can be badly violated already in the Standard Model framework, while a similar relation for  $D_s^+$  radiative decays, i.e.

$$R_K \equiv \frac{\Gamma(D_s^+ \rightarrow K^{*+}\gamma)}{\Gamma(D_s^+ \rightarrow \rho^+\gamma)} = \tan^2\theta_c \quad (2)$$

offers a much better test for  $c \rightarrow u\gamma$ .

The paper is organised as follows: in Sect. 2 we sketch the relevant theoretical framework for radiative decays; in Sect. 3 we give results for the branching ratios of the widths for Cabibbo suppressed and Cabibbo allowed radiative decays; finally we make a short discussion of our results in Sect. 4.

## 2 Theoretical framework

Experimentally radiative decays of  $D$  mesons have not yet been measured, while the known branching ratios of  $D^*$  radiative decays [10, 11] can be described using the combination HQET + CHL [8, 12, 13].

The initial HQET ideas [14, 15] were implemented with the chiral Lagrangian formalism for light pseudoscalar mesons first in [4, 5, 6], and the electromagnetic interaction included in [12, 13, 16]. Consequently, the light vector mesons were introduced [7], following the hidden symmetry approach [17]. We will follow the model described in [8], where in addition to [7] the electromagnetic (EM) interaction was introduced.

Let us briefly describe the relevant terms (for the charm meson radiative weak decays) of the Lagrangian [8]. The main contribution comes from the

odd-parity Lagrangian

$$\begin{aligned}
\mathcal{L}_{odd} = & -4e\sqrt{2}\frac{C_{V\pi\gamma}}{f}\epsilon^{\mu\nu\alpha\beta}Tr(\{\partial_\mu\rho_\nu, \Pi\}Q\partial_\alpha B_\beta) \\
& -4\frac{C_{VV\Pi}}{f}\epsilon^{\mu\nu\alpha\beta}Tr(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\Pi) \\
& -\lambda'eTr[H_a\sigma_{\mu\nu}F^{\mu\nu}(B)\bar{H}_a] \\
& +i\lambda Tr[H_a\sigma_{\mu\nu}F^{\mu\nu}(\hat{\rho})_{ab}\bar{H}_b], 
\end{aligned} \tag{3}$$

where  $C_{VV\Pi} = 0.423$ ,  $C_{V\pi\gamma} = -3.26 \times 10^{-2}$ [18, 19],  $f = 132$  MeV is the pseudoscalar decay constant, while the phenomenological parameters  $\lambda$  and  $\lambda'$  are constrained by the analysis [8]:

$$|\lambda' + \frac{2}{3}\lambda| = (0.50 \pm 0.15) \text{ GeV}^{-1}, \tag{4}$$

$$|\lambda' - \frac{\lambda}{3}| < 0.19 \text{ GeV}^{-1}. \tag{5}$$

In (3)  $\Pi$  and  $\rho_\mu$  are the usual pseudoscalar and vector Hermitian matrices

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}, \tag{6}$$

$$\rho_\mu = \begin{pmatrix} \frac{\rho_\mu^0 + \omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{-\rho_\mu^0 + \omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \Phi_\mu \end{pmatrix} \tag{7}$$

with  $\eta = \eta_8 \cos \theta - \eta_0 \sin \theta$ ,  $\eta' = \eta_8 \sin \theta + \eta_0 \cos \theta$  and  $\theta = -23^\circ$  [10] is the  $\eta - \eta'$  mixing angle.  $Q = diag(2/3, -1/3, -1/3)$  is the light quark sector charge matrix,

$$H_a = \frac{1}{2}(1+\not{v})(\sqrt{m_{D^{a*}}}D_\mu^{a*}\gamma^\mu - \sqrt{m_{D^a}}D^a\gamma_5) , \quad (8)$$

where  $D_\mu^{a*}$  and  $D^a$  annihilate, respectively, a spin-one and spin-zero meson  $c\bar{q}^a$  ( $q^a = u, d$  or  $s$ ) of velocity  $v_\mu$  and  $\bar{H}_a \equiv \gamma^0 H_a^\dagger \gamma^0$ . Finally,  $F_{\mu\nu}(\hat{\rho}) = \partial_\mu \hat{\rho}_\nu - \partial_\nu \hat{\rho}_\mu + [\hat{\rho}_\mu, \hat{\rho}_\nu]$ ,  $\hat{\rho}_\mu = ig_V \rho_\mu / \sqrt{2}$  with  $g_V = 5.8$  [7], and  $F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu$  with  $B_\mu$  being the photon field with the EM coupling constant  $e$ .

The first (third) term in (3) describes the anomalous direct emission of the photon by the light (heavy) meson, while the second (fourth) term, together with the vector meson dominance (VMD) coupling

$$\mathcal{L}_{V-\gamma} = -m_V^2 \frac{e}{g_V} B_\mu (\rho^{0\mu} + \frac{1}{3}\omega^\mu - \frac{\sqrt{2}}{3}\Phi^\mu) \quad (9)$$

describes a two step photon emission, with an intermediate neutral vector meson with mass  $m_V$  which transforms to the final photon.

A charged charm meson can emit a real photon also through the usual electromagnetic coupling

$$\mathcal{L}_{EM} = -ev^\mu B_\mu Tr[H_a(Q - 2/3)_{ab}\bar{H}_b] , \quad (10)$$

while a charged light vector meson can produce through

$$\mathcal{L}_{VVV} = \frac{1}{2g_V^2} tr[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})] \quad (11)$$

first a neutral vector meson, which subsequently transforms via VMD (9) to a photon.

The weak Lagrangian is approximated by the current-current type interaction

$$\mathcal{L}_W^{eff}(\Delta c = 1) = -\frac{G_F}{\sqrt{2}}[a_1(\bar{u}d')^\mu(\bar{s}'c)_\mu + a_2(\bar{s}'d')^\mu(\bar{u}c)_\mu] , \quad (12)$$

where  $(\bar{q}_1 q_2)^\mu \equiv \bar{q}_1 \gamma^\mu (1 - \gamma^5) q_2$ ,  $G_F$  is the Fermi constant and  $a_{1,2}$  are the QCD Wilson coefficients, which depend on the energy scale  $\mu$ . One expects the scale to be the heavy quark mass and we take  $\mu \simeq 1.5$  GeV which gives  $a_1 = 1.2$  and  $a_2 = -0.5$ , with an approximate 20% error. Since we are interested only in the physics of the first two generations, we can express the weak eigenstates  $d'$ ,  $s'$  with the mass eigenstates  $d$ ,  $s$  using the Cabibbo angle instead of the CKM matrix:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & -\sin \theta_c \\ \sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad (13)$$

with  $\sin \theta_c = 0.222$ . Possible contributions caused by the penguin type diagrams are found to be very small [9].

Many heavy meson weak nonleptonic amplitudes [20, 21, 22] have been calculated using the factorization approximation. In this approach the quark currents are approximated by the “bosonised” currents [4, 7, 8], of which only

$$(\bar{q}^a c)_\mu = i(m_{D^{*a}} f_{D^{*a}} D_\mu^{*a} - m_{D^a} f_{D^a} v_\mu D^a) , \quad (14)$$

$$(\bar{q}_b q_a)^\mu = -f \partial^\mu \Pi_{ab} + m_V f_V \rho_{ab}^\mu \quad (15)$$

will contribute to our amplitudes. The numerical values for the masses will be taken from [10] and for the decay constants from [22].

It is now straightforward to calculate the decay widths. The result, of course, depends on the numerical values we take for  $(\lambda' + 2\lambda/3)$  and  $(\lambda' - \lambda/3)$ .

### 3 Cabibbo suppressed radiative weak decays in HQET + CHL

Apart from the Cabibbo allowed decays  $D^0 \rightarrow \bar{K}^{*0}\gamma$  and  $D_s^+ \rightarrow \rho^+\gamma$ , five once Cabibbo suppressed ( $D^0 \rightarrow \rho^0\gamma$ ,  $D^0 \rightarrow \omega\gamma$ ,  $D^0 \rightarrow \phi\gamma$ ,  $D^+ \rightarrow \rho^+\gamma$ ,  $D_s^+ \rightarrow K^{*+}\gamma$ ) and two doubly Cabibbo suppressed ( $D^0 \rightarrow K^{*0}\gamma$  and  $D^+ \rightarrow K^{*+}\gamma$ ) decays are possible.

According to [21] the weak amplitudes can be categorized into two groups: quark decays and weak annihilations. As these authors have noticed, the factorization works much better for the quark decays. The decays of  $D_s^+$  and  $D^+$  are of the quark decay type (their amplitudes are proportional to  $a_1$ , see below), and therefore the results for these decays are more trustworthy than for the  $D^0$   $D$  decays, which proceed through weak annihilation diagrams (proportional to  $a_2$ ).

We write the amplitude for the  $D^q \rightarrow V^q\gamma$ , where  $q$  stands for the charge of D meson ( $q = 1$  stands for + charge, while  $q = 0$  is for the neutral D mesons)

$$A(D^q \rightarrow V^q\gamma) = e \frac{G_F}{\sqrt{2}} K_c a(q) [C_{DV\gamma}^{(1)} \epsilon_{\mu\nu\alpha\beta} k^\mu \epsilon_\gamma^{\nu*} v^\alpha \epsilon_V^{\beta*} + i C_{DV\gamma}^{(2)} m_V (\epsilon_\gamma^* \cdot \epsilon_V^* - \frac{\epsilon_\gamma^* \cdot p_V \epsilon_V^* \cdot k}{p_V \cdot k})] \quad (16)$$

with  $a(+1) = a_1$  and  $a(0) = a_2$ .  $(k, \epsilon_\gamma)$  and  $(p_V, \epsilon_V)$  are the 4-momenta and polarization vectors of the photon and vector meson respectively, while  $v$  is the 4-velocity of the heavy meson.

The overall factor  $K_c$  contains the Cabibbo angle and is equal to  $\cos \theta_c$ <sup>2</sup> for allowed decays, to  $+\sin \theta_c \cos \theta_c$  (when there is no  $s$  quark or antiquark in the final  $V$ ) or  $-\sin \theta_c \cos \theta_c$  (when there is at least one  $s$  quark or antiquark in the final  $V$ ) for once suppressed decays and to  $-\sin \theta_c$ <sup>2</sup> for double suppressed decays. The coefficients  $C^{(i)}$  in (16) can be written as

$$\begin{aligned} C_{DV\gamma}^{(1)} &= (C_{VV\Pi} \frac{1}{g_V} + C_{V\Pi\gamma}) 4\sqrt{2} f_D m_D^3 b^V \\ &+ 4[\lambda' + (\frac{2}{3} - q)\lambda] f_{D^*} f_V \frac{m_{D^*} m_V}{m_{D^*}^2 - m_V^2} \sqrt{m_D m_{D^*}} b_0^V , \end{aligned} \quad (17)$$

$$C_{DV\gamma}^{(2)} = q f_D f_V . \quad (18)$$

The coefficient  $b^V$  is equal to  $(2/3 - q)/(m_D^2 - m_P^2)$  for  $V = (\bar{K}^{*0}, K^{*0}, \rho^+, K^{*+})$ , for which  $P = (\bar{K}^0, K^0, \pi^+, K^+)$ . For the remaining final state vector mesons this coefficient is expressed as

$$b^V = \sum_{i=1}^3 \frac{b^{VP_i}}{m_D^2 - m_{P_i}^2} , \quad (19)$$

where the pole coefficients  $b^{VP_i}$  are given in Table 1. Furthermore we have  $b_0^V = -1/\sqrt{2}$  for  $V = \rho^0$ ,  $b_0^V = 1/\sqrt{2}$  for  $V = \omega$  and  $b_0^V = 1$  otherwise.

In ref. [1, 2] it was noticed that a nice bonus can be obtained by measuring the charm meson decay width  $D \rightarrow \rho/\omega\gamma$  which is generated by  $c \rightarrow u\gamma$  transitions. Namely, the authors claim that observing the violation of equation (1) would then eventually signal New Physics [1]. Using our model, which pretends to describe the low energy meson physics within the Standard Model, we find that this relation does not exactly hold due to U(3) breaking. We assume that the leading effect of this breaking is to change the

values of the masses and decay constants for different members of the same multiplets and between octet and singlet. However, one would naively expect deviations from this limit in the Standard Model of the order of 20 – 30%. We will see that this is not true for the  $D^0 \rightarrow \rho^0/\omega\gamma$  decay, but it is correct for  $D_s^+$  Cabibbo suppressed radiative weak decay.

Within our framework  $\lambda$  and  $\lambda'$  are the most important parameters for charm meson radiative decays [8], and therefore we present the ratios of the decay widths as functions of combinations of  $\lambda$  and  $\lambda'$ . Our result for  $R_\rho$  (1) is showed on Fig. 1: if the combination of  $(\lambda' + \frac{2}{3}\lambda)$  turns out to be negative, the ratio  $R_\rho$  can approach 0. As it is known from  $D^0 \rightarrow \bar{K}^{*0}\gamma$  [8], the negative values  $(\lambda' + \frac{2}{3}\lambda)$  cause a destructive interference between the photon emission by the heavy meson and the photon emission by the light meson. A similar effect is possible also in the decay  $D^0 \rightarrow \rho^0\gamma$ , only that the 0 is achieved at a different value of  $(\lambda' + \frac{2}{3}\lambda)$ , because the model parameters are here slightly different due to  $U(3)$  breaking. It is obvious that such a large sensitivity to the model parameters does not allow us to conclude anything about some New Physics. If  $(\lambda' + \frac{2}{3}\lambda)$  turns out to be positive, the decays are much easier to detect experimentally, and also the theoretical situation is clearer, since the curve is approaching the ideal theoretical value. A large disagreement with the theoretical prediction (1) would give in this case some sign of New Physics. But even here one should be careful, since in this case the amplitudes are approximately proportional to the decay constants of the final vector meson. This can be seen, if we calculate the decay  $D^0 \rightarrow \omega\gamma$  with the values of the light vector decay constants taken from [22] :  $f_{K^*} = f_\rho = 221$  MeV and  $f_\omega = 156$  MeV. In this case we get for  $R_\omega$  a similar curve as in Fig.

1, but for large positive values  $(\lambda' + \frac{2}{3}\lambda)$  the ratio is approaching a value of approximately 0.5 instead of 1. The fact can be explained by the difference in the decay constants, i.e.  $(f_\omega/f_{K*})^2 \simeq 0.5$ .

The ratio  $\Gamma(D^0 \rightarrow \Phi\gamma)/\Gamma(D^0 \rightarrow \bar{K}^{*0}\gamma)$  would indicate the deviation from  $\tan \theta_c^2$  instead of  $\tan \theta_c^2/2$  like for the  $\rho, \omega$  case. When calculated, it exhibit a similar behaviour like  $D^0 \rightarrow \omega\gamma$ , and therefore we find it is not useful to understand  $c \rightarrow u\gamma$  physics.

The decays  $D^+ \rightarrow \rho^+\gamma$  is also not of great importance in our purpose to find New Physics, since the  $D^+$  does not have Cabibbo allowed decays.

Contrary to the above cases we find that the decay  $D_s^+ \rightarrow K^{*+}\gamma$  offers a much better chance to test New Physics. Using the general formulas for the amplitudes (16) it is easy to derive a deviation from equation (2), which is exactly correct only in the U(3) limit. The result for  $R_K$  as a function of  $(\lambda' - \frac{1}{3}\lambda)$  is presented on Fig. 2 (note the changed scale with respect to Fig. 1). We notice that the result is rather stable within the allowed region for  $(\lambda' - \frac{1}{3}\lambda)$ . The discrepancy to relation (2) is due to U(3) breaking and is of order 30%, as usually expected. If the the experimental results are found to be far away from the curve Fig. 2, one can interpret it as a sign of New Physics.

We point out that it is difficult to observe all these decays. In fact the Cabibbo allowed decays are already rare: the branching ratio for  $D^0 \rightarrow \bar{K}^{*0}\gamma$  is smaller than  $0.3 \times 10^{-4}$  for  $(\lambda' + 2\lambda/3) < 0$  and around  $(2 - 4) \times 10^{-4}$  for  $(\lambda' + 2\lambda/3) > 0$ , while for  $D_s^+ \rightarrow \rho^+\gamma$  the branching ratio is around  $(2 - 7) \times 10^{-4}$  [8].

## 4 Conclusions

We determine amplitudes of Cabibbo suppressed radiative decays using the combination of heavy quark symmetry and chiral symmetry, which builds an effective strong, weak and electromagnetic Lagrangian. This theoretical framework just illustrates the characteristics of these amplitudes in the Standard Model. In our framework two parameters,  $\lambda$  and  $\lambda'$ , are not well known. We show the dependence of the ratio between the Cabibbo suppressed and Cabibbo allowed decay widths on the combination of  $\lambda$  and  $\lambda'$ . We find that it is better to search for a signal of New Physics coming from  $c \rightarrow u\gamma$  decays from the ratio  $\Gamma(D_s^+ \rightarrow K^{*+}\gamma)/\Gamma(D_s^+ \rightarrow \rho^+\gamma)$  instead of the proposed ratio  $\Gamma(D^0 \rightarrow \rho^0/\omega\gamma)/\Gamma(D^0 \rightarrow \bar{K}^{*0}\gamma)$  [1, 2, 3].

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## FIGURES

Fig. 1: The ratio  $2R_{\rho/\omega}/\tan \theta_c^2$  as function of the combination  $\lambda' + 2\lambda/3$ . The full (dashed/dot-dashed) lines denote the experimentally allowed (forbidden) values for this combination. In the U(3) symmetry limit of the Standard Model this ratio is equal 1.

Fig. 2: The ratio  $R_K/\tan \theta_c^2$  as function of the combination  $\lambda' - \lambda/3$ . The full (dashed) lines denote the experimentally allowed (forbidden) values for this combination. In the U(3) symmetry limit of the Standard Model this ratio is equal 1.

	$\pi^0$	$\eta$	$\eta'$
$\rho^0$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{2}}c(c - \sqrt{2}s)$	$-\frac{1}{\sqrt{2}}s(\sqrt{2}c + s)$
$\omega$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}c(c - \sqrt{2}s)$	$-\frac{1}{3\sqrt{2}}s(\sqrt{2}c + s)$
$\phi$	0	$\frac{\sqrt{2}}{3}c(\sqrt{2}c + s)$	$-\frac{\sqrt{2}}{3}s(c - \sqrt{2}s)$

Table 1: The  $b^{VP_i}$  coefficents defined in relation (19), where  $s = \sin \theta$ ,  $c = \cos \theta$  and  $\theta$  is the  $\eta - \eta'$  mixing angle.

Fig. 1

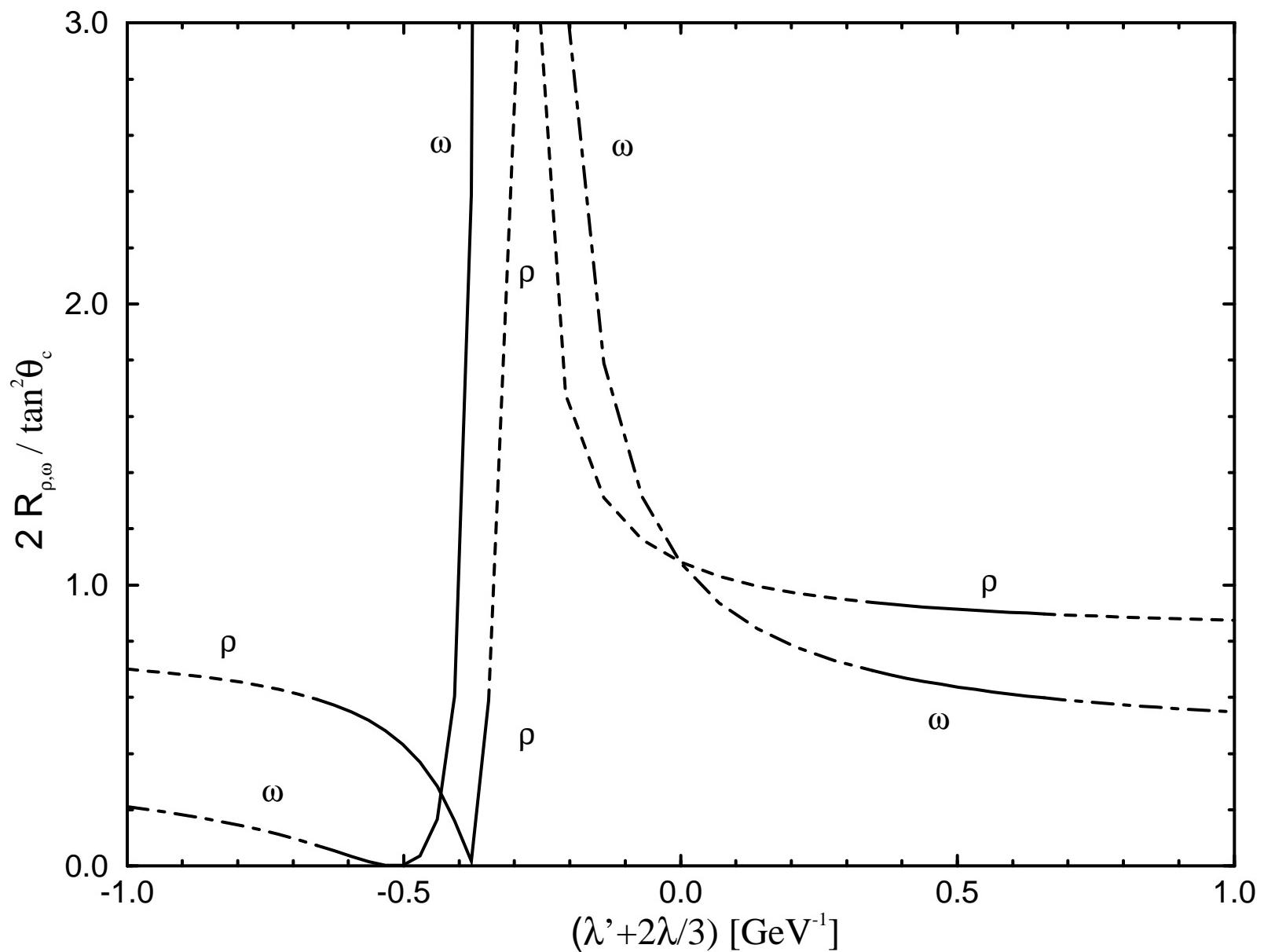


Fig. 2

